Lesson Title	Projectile Motion
Length of Lesson	2-3 Days
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Subject	Physics
Grade Level	11-12
State Standards	Physics: 1g, 2d
DOK Level	DOK 4
DOK Application	Synthesis, Create, Construct, Estimate, Predict
National Standards	9-12: B (Physical Science)
Graduate Research Element	Robotics, Computer Programming

Student Learning Goal:
Students will be able to predict how elevation angle affects with projectile motion. They will be able to design and build a simple robot that can fire a projectile at a target.

Physics: 1g. Collect, analyze, and draw conclusions from data to create a formal presentation using available technology (e.g., computers, calculators, SmartBoard, CBL's, etc.)
2d. Apply the effects of the universal gravitation law to graph and interpret the force between two masses, acceleration due to gravity, and planetary motion.

- Situations where g is constant (falling bodies)

National Science Education Standards of Content 9 – 12
B (Motions and Forces): Gravitation is a universal force that mass exerts on any other mass. The strength of the gravitational attractive force between two masses is proportional to the masses and inversely proportional to the square of the distance between them.

Materials Needed (supplies, hand-outs, resources):
Lego MindStorms, writing utensils and paper (for solving problems).
Optional: PowerPoint, videos of projectiles.

Lesson Performance Task/Assessment:
The students will be able to solve written problems involving projectile motion.

The students will compete by programming their robots to hit targets at known ranges with a single shot. This will ensure that the students must calculate angle of elevation and initial velocity, and be able to accurately aim their robot.

Lesson Relevance to Performance Task and Students:

Students will be able to understand vector components in projectile motion, learn how to design mechanical systems, make simple programs. The students will demonstrate their skill by hitting a stationary target.

Anticipatory Set/Capture Interest:
The Lego Marines want a new cannon that can take out the enemy base. The cannon will be aimed manually, but they want elevation to be set automatically via a program. Since the enemy base is well defended, if the cannon misses with the first shot, it will be targeted and destroyed. The students will be divided into design teams, and they will be provided with a Lego MindStorms kit. It is the job of the students to design and demonstrate their cannon for the Lego Marines. The teacher will play the role of deciding what cannon design the Lego Marines should buy.

Guided Practice:
The teacher should start by teaching about projectile motion. PowerPoint presentations and videos can be useful. The students should learn that the horizontal and vertical motions are independent of each other and that both are dependent on the initial velocity and angle of elevation. The equations for the motion of the projectile are generally expressed as:

$$D_{verticle}(t) = \frac{1}{2}gt^2 + sin(\theta)v_{initial}t + D_{verticle_{initial}}, \quad (1)$$

and

$$D_{horizontal}(t) = cos(\theta)v_{initial}t. \quad (2)$$

From these equations, we can get the following equations that describe the projectile's motion:

$$0 = \frac{1}{2}gt^2 + \sin(\theta)v_{initial}t - h, \quad (3)$$

and

$$R = \cos(\theta)v_{initial}t. \quad (4)$$

The variable h is the height of the target relative to the muzzle of the cannon, R is the range of the target, g is the gravitational constant, $v_{initial}$ is the muzzle velocity, and θ is the angle of elevation. In these two equations, θ is the only unknown. We can solve for θ.

(from 4) $$t = \frac{R}{\cos(\theta)v_{initial}} \quad (5)$$

(substitute for *t* in 3) $$0 = \frac{1}{2}g\left[\frac{R}{\cos(\theta)v_{initial}}\right]^2 + \sin(\theta)\, v_{initial}\left[\frac{R}{\cos(\theta)v_{initial}}\right] - h \quad (6)$$

(simplify 6) $$0 = \frac{gR^2}{2{v_{initial}}^2}\sec^2(\theta) + R\tan(\theta) - h \quad (7)$$

$$\text{Let } A = \frac{gR^2}{2{v_{initial}}^2} \quad (8)$$

(substitute A in 7) $$0 = A\sec^2(\theta) + R\tan(\theta) - h \quad (9)$$

(substitute trig identity) $$0 = A[\tan^2(\theta) + 1] + R\tan(\theta) - h \quad (10)$$

(simplify 10) $$0 = A\tan^2(\theta) + R\tan(\theta) + (A - h) \quad (11)$$

(quadratic formula) $$\tan(\theta) = \frac{-R \pm \sqrt{R^2 - 4(A)(A-h)}}{2A} \quad (12)$$

(inverse *tan*) $$\theta = \tan^{-1}\left(\frac{-R \pm \sqrt{R^2 - 4(A)(A-h)}}{2A}\right) \quad (13)$$

This derivation should be shown to the students, but it is not necessary for them to remember every step. At this point, the students should work through one or two problems so that they can see that the equation does indeed work.

At this point, the teacher should begin talking about ways to measure the muzzle velocity of the cannon. The teacher could start by asking the students for ideas about how to do this. There are many options for doing this, and it can be computed by firing the cannon at any angle of elevation. The teacher does not have to tell the students, but the easiest methods are one of the following three methods.

1. Fire the cannon in a level configuration from a known height above the ground, and measure the distance the ball travels. The time it takes for the ball to travel that distance can be computed by $t = \sqrt{\frac{-2h}{g}}$, where h is the height of the cannon.
2. Fire the cannon with an elevation of 90 degrees, and measure the maximum height above the muzzle the ball reaches. The speed can be computed using the potential and kinetic energy equations by making $U = E_k$ and solve for v. (Note that h is the height of the ball here.)

   Potential Energy: $$U = mgh \quad (14)$$

   Kinetic Energy: $$E_k = \frac{1}{2}mv^2 \quad (15)$$
3. The third solution is to fire the cannon at any angle and measure the distance traveled by the ball. After a lot of trig and algebra, it turns out that the answer can be computed by the equation below. (The variable h again is the height of the target.)

$$v_{initial} = \sqrt{\frac{gR^2}{2\cos^2(\theta)(h - \tan(\theta)R)}} \quad (16)$$

The first method is probably the easiest and most accurate. The second method can be modified to work at any angle as long as the student realizes their result will be $v = sin(\theta)v_{initial}$. A benefit of the second method is that the students can apply what they know about potential and kinetic energy to solving the problem. The third solution can be as accurate as the first, but requires some more math, so most students will probably not use this technique.

The teacher should let the students think about how they will measure their cannon's muzzle velocity while they are building it. They will have a lot of time to think about it. The teacher can easily lead the students to the first method by either talking about the time it take an object to free fall to the ground or second method by talking about energy.

Independent Practice:

The students will have time to build, program, and test their cannon. Since the students only need to control elevation, this task should take 30-60 minutes. The students should also be provided with one ball and reminded that they must hit the target on the first shot.

If the students get to start building one day and finish the next, it might not be a bad thing.

Remediation and/or Enrichment:
Remediation: individual IEP; partner help throughout the lesson; the teacher can observe the students and intervene during the independent practice.

Check(s) for Understanding:
Can the student's cannon hit the target with a single shot?
If the range is changed, can they recalculate and hit the target with the first shot?
Do they know when the target is out of range?
If they miss, can they calculate what the angle should have been?
Depending on the conditions, there might be a lot of randomness in their cannon's performance. Can they explain why their cannon is not accurate?

Extra Step:
There are usually two elevation angles that will result in a hit. Can the students demonstrate this?

Closure:
Ask the students what they could have done to improve their accuracy. Possible answers could be:

- compensate for wind,
- compensate for air resistance,
- compensate for a change in muzzle height when the elevation angle is changed,
- change the gear ratios to give them finer control over the elevation angle,
- move closer to the target.

Possible Alternate Subject Integrations:
Math, Robotics, Computer Programming

Teacher Notes:
This lesson assumes that the students have experience with the Lego MindStorms robot. If this is not the case, the time it takes to teach this lesson can be longer than anticipated.