Lesson Title	GPS Clocks and Relativity
Length of Lesson	3 Days
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Subject	Physics
Grade Level	11-12 (Physics)
State Standards	Physics: 1a, g; (Advanced Enrichment)
DOK Level	DOK 4
DOK Application	Design, Apply Concepts, Analyze
National Standards	9 – 12: B (physical); E (technology)
Graduate Research Element	Geological applications of GPS

Student Learning Goal: This lesson was prepared for a 12th grade survey of modern physics class taught at the Mississippi School for Mathematics & Science; however, it could be adapted for use in a general physics course at the 11-12 grade level. This lesson is designed to provide a DOK 4 level of understanding for Mississippi State Science Standards Physics: 1a, g; with advanced enrichment.

Physics: 1. Apply inquiry-based and problem-solving processes and skills to scientific investigations: (a) Use current technologies such as CD-ROM, DVD, Internet, and on-line data search to explore current research related to a specific topic; (g) Collect, analyze, and draw conclusions from data to create a formal presentation using available technology (e.g., computers, calculators, SmartBoard, CBL's, etc.)

National Science Education Standards of Content 9 – 12
A (Inquiry): identify questions and concepts that guide scientific investigations.

E: (Science and Technology): abilities of technological design; understanding about science and technology

Materials Needed (supplies, hand-outs, resources): Classroom handout on special relativity calculations, access to a programming language of choice, access to Microsoft PowerPoint

Lesson Performance Task/Assessment: Students analyze and explain concepts of special and general relativity as applied to the engineering of Satellite clocks onboard satellites, e.g., GPS satellites.

Student teams learn to calculate the special and general relativistic corrections that are necessary for engineering design of an on-board clock for GPS satellites.

Student teams use a programming language of choice to generate a set of calculations for a range of orbital altitudes for a satellite clock.

Student teams prepare and present Microsoft PowerPoint presentations explaining the design of their GPS clocks for onboard satellites.

Lesson Relevance to Performance Task and Students:
These lessons and performance tasks will increase students' interest in the subject through the use of technology resources (programming language, PowerPoint, internet) to generate a data set and create a presentation of their design process and result.

Anticipatory Set/Capture Interest:
The teacher will introduce to students some sources for GPS error, specifically the near 100 meter corrections to GPS engineering design due to relativistic effects.

Guided Practice:

Day One: Student teams are determined by the instructor and the calculations are modeled using internet gathered data on some existing satellite of choice. The necessary theory is as follows:

Special Relativity (SR) predicts that fast moving clocks will appear to run slower than stationary clocks. General Relativity (GR) predicts that clocks closer to the planet's center of mass will run slower because such clocks are located in a stronger gravitational field.

The on-board clocks of satellites (e.g., GPS clocks) must be engineered to compensate for the effects of both SR and GR. In practice, engineers reset the rate in which satellite clocks tick to compensate for SR and GR before launch. While the clocks are observed to run at the corrected rates (slower or faster) before launch, the clocks will be observed to run again at the synchronized rate when orbiting at altitude relative to the ground-based, stationary clocks.

The SR multiplier depends upon velocity (v) and the GR multiplier depends upon displacement (r) from the planet's center of mass,
where:

Speed of Light: c = 2.99 x 10^8 meters/second

Gravity Constant: G = 6.67 x 10-11 Newton·meter^2/kilogram^2

Mass of Earth: M = 5.98 x 10^{24} kilograms

$$\gamma(v) = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c} \qquad \gamma(r) = \sqrt{\frac{1-2\cdot GM}{c^2\cdot r}}$$

Binomial expansions of both the SR and GR are needed to combine the two effects as factors where velocities are << speed of light.

$$\gamma(v) \approx 1+\frac{\beta^2}{2} \qquad \gamma(r) \approx 1-\frac{GM}{c^2\cdot r}$$

So that the combined multiplier for both SR and GR becomes:

$$\gamma(v)\cdot\gamma(r)\approx 1+\frac{\beta^2}{2}-\frac{GM}{c^2\cdot r}-\frac{GM\cdot\beta^2}{2\cdot c^2\cdot r}$$

For orbits of low eccentricity (nearly circular), orbital altitudes above the planetary center of mass and orbital velocities can be calculated by placing Newton's laws of centripetal force and gravitational force into equilibrium, and solving for the needed variable. Also, velocity is the ratio of distance traveled in the near circular orbit (2πr in meters) to the period of orbit (T) in seconds.

$$\frac{v^2}{r}\approx\frac{GM}{r^2}\qquad v\approx\frac{2\pi\cdot r}{T}$$

Independent Practice:

Day Two:
Student teams will use any software of choice to program calculations for low-eccentricity orbits. The code must generate time-dilation corrections per orbit. That is, for any desired range of orbital altitudes, the program code will generate the SR·GR multipliers and multiply by the period of orbit (T) observed by a ground clock. In sum, the student teams generate satellite clock corrections that engineers need for any near-circular satellite orbit.

Day Three:
As advanced enrichment, it is recommended that student teams prepare PowerPoint presentations of their design process and results. Next the PowerPoint presentations should be practiced and finally delivered before a panel of teachers at a convenient date.

Remediation and/or Enrichment: individual IEP; partner help throughout lesson; shorten parts of assignment; focus upon smaller elements of the process

Enrichment/Extension:

1. Set up a seminar which includes students from other science classrooms as an audience for the student team design presentations.
2. Calculus-based physics classrooms may develop the concept and process of binomial expansion of functions into Taylor Series.

Check(s) for Understanding:

Day One: Do the students understand how to calculate, velocity, period, and time-dilations for the orbiting clock?

Day Two: Do the students understand how to generate time-dilations for a range of orbital altitudes using their programming language of choice?

Day Three: Do the students know how to employ PowerPoint to relate the process or method they used to generate their data and to present their results?

Closure:
Day One: Have the student teams generate one time-dilation computation for one orbit of the Hubble Space Telescope. Then have the student teams compare their answers.

Day Two: Have the student teams explain the advantages of their choice of programming environment/ language.

Day Three: Have the panel of teachers offer suggestions for improvement for student presentations.

Possible Alternate Subject Integrations:
*Math – can manipulate mathematical expressions to isolate needed variables

*Programming/Computer Science – can use a programming language to generate computations over a range of variables

*Language Arts – can use PowerPoint to deliver a public presentation of an engineering design

Teacher Notes:
* Calculus-based physics classrooms may develop the concept and process of binomial expansion of functions into Taylor Series.