Lesson Title	Folding Fractions
Length of Lesson	1 50-minute lesson
Created By	Sean Owens
Subject	Mathematics, Geometry
Grade Level	9^{th} – 12^{th}
State Standards	9^{th} - 12^{th} Pre-Algebra: 4 a; 4 b; 4c Transition to Algebra: 2 j; 3 b; 4 a; 5 a Algebra I: 4 a Geometry: 2 a; 3 a Algebra II: 1 e
DOK Level	DOK 1 – Pre-Algebra DOK 2 – Pre-Algebra, Transition to Algebra, Algebra, Geometry DOK 3 – Transition to Algebra, Geometry, Algebra II
DOK Application	1 – Calculate, Measure, Tabulate 2 – Infer, Identify Patterns, Predict, Interpret, Make Observations, Relate 3 – Develop a Logical Argument, Compare, Hypothesize, Formulate
National Standards	9-12: B: Geometry C: Measurement D: Data Analysis and Probability
Graduate Research Element	This lesson looks at mathematical sequences through the folding of paper and looking at the relationships between the areas of each successive fold. In my research, I deconstruct audio signals that are represented as sequences of audio samples. Furthermore, the concept of area is a large portion of my research, as I am working to fit a hardware circuit system of a certain physical area onto a chip that has a different physical area.

Student Learning Goal:
After performing this lesson, students will be able to apply data analysis techniques to derive a mathematical sequence describing a physical event.

This lesson will address Mississippi 9-12 Mathematics standards: Pre-Algebra 4a, 4b, and 4c; Transition to Algebra 2j, 3b, 4a, 5a; Algebra 1 4a; and Geometry 2a and 3a. It will also address National 9-12 Mathematics standards B, C, and D.

State Standards: 9th – 12th Mathematics

Pre-Algebra – 4a: Solve real-world application problems that include length, area, perimeter, and circumference using standard measurements.

Pre-Algebra – 4b: Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Pre-Algebra – 4c: Use formulas and/or appropriate measuring tools to find length and angle measures (to appropriate levels of precision), perimeter, area, volume, and surface area of polygons, circles, spheres, cones, pyramids, and composite or irregular figures.

Transition to Algebra – 2j: Apply ratios and use proportional reasoning to solve real-world algebraic problems.

Transition to Algebra – 3b: Apply proportional reasoning to determine similar figures and find unknown measures.

Transition to Algebra – 4a: Solve real-world problems involving measurements (i.e., circumference, perimeter, area, volume, distance, temperature, etc.).

Transition to Algebra – 5a: Construct graphs, make predictions, and draw conclusions from tables, line graphs, and scatter plots.

Algebra I – 4a: Solve real-world problems involving formulas for perimeter, area, distance, and rate.

Geometry – 2a: Represent data from geometric and real-world contexts with expressions, formulas, tables, charts, graphs, relations, and functions.

Geometry – 3a: Use inductive reasoning to make conjectures and deductive reasoning to make valid conclusions.

Algebra II – 1e: Solve applications and problems in mathematical settings involving arithmetic and geometric sequences and series.

National Standards: 9th – 12th Mathematics

B (Geometry):

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Apply transformations and use symmetry to analyze mathematical situations.

C (Measurement):

- Apply appropriate techniques, tools, and formulas to determine measurements.

D (Data Analysis and Probability)

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.
- Develop and evaluate inferences and predictions that are based on data.

Materials Needed (supplies, hand-outs, resources):

- Paper (8 ½ x 11 or square)
- Handout (See fractions_handout.doc)
- Rulers

Lesson Performance Task/Assessment:
Students will successively fold paper and measure the dimensions of the resulting rectangles. The students will record this data and the computed areas in a table. The students will analyze the data collected and hypothesize an expression for the relation of the areas of the paper between numbers of folds. The students will use expression to predict future results.

Lesson Relevance to Performance Task and Students:
The students will fold and measure area data of a piece of paper to demonstrate and understanding of data collection and analyze the data collected to show their ability to interpret data. The students will hypothesize an expression for the relationship between areas to exhibit their understanding of geometric progression and proportional relationships. Finally, students will use a mathematical expression of a physical event to predict the outcome of future events demonstrating their understanding of mathematical series.

Anticipatory Set/Capture Interest:
To begin this lesson, the instructor should ask, "Is it possible to fold a piece of paper in half 7 times?" The instructor can then explain how without help it is impossible to fold a piece of paper 7 times. The instructor can then ask, "Why do you think this is the case?" After, the instructor can lead into the guided practice.

Guided Practice:
During the guided practice, the instructor should give the instructions for the activity: The students should fold the paper in half, alternating the direction each fold, and measure and record the length, width, and area of the paper after each fold for as many folds as they can. The instructor can lead the students through the first fold and data entry if necessary. Then the instructor will tell the students to analyze the data to figure out the relationship between the areas. The instructor will then lead into the independent practice section.

Independent Practice:
During the independent practice section, the students will fill out the table provided in the handout (see fractions_handout.doc) and develop an expression for relating the areas at each fold. The instructor will observe the process and provide individual help as needed. After the appropriate amount of time, the instructor can lead into the discussion and closure section of the lesson.

Remediation and/or Enrichment:
Remediation: Individual IEP
Enrichment: Have the student use the expression developed to compute other facts relating to future folds (e.g., how many layers thick would the paper be after x number of folds?)

Check(s) for Understanding:
What is the formula for the area of a square? What is the ratio of the current area to the previous area? What should the next area be given your sequence?

Closure:
In the discussion and closure section of the lesson, the instructor can poll the students for their data to develop a "class-wide" data set. From this the instructor can ask the students to present their theories on the relationship between the areas after each fold. Once the relationship has been established, the instructor can explain how this relationship can be written as a sequence and how that can be used to predict future values. The instructor can then discuss how sequences are used in audio compression such as mp3s by predicting values based on a compressed set of the original sequence.

Possible Alternate Subject Integrations:
Algebra

Teacher Notes:

- Note that measuring the rectangle will become increasing difficult as the number of folds increases. Using a standard 8 ½ x 11 sheet of paper leads to 1/16 of an inch resolution after only 4 folds. Therefore it is recommended to start with a paper size of even units to ensure that measurements can be made with relative expectation that the measurement can be rounded without inducing error in the data. Also, instructing the students that this possibility for error exists can be helpful.