Lesson Title	Math Sequences with Digital Logic Circuits
Length of Lesson	50 minutes
Created By	Sean Owens
Subject	Mathematics, Algebra II
Grade Level	10^{th} – 12^{th}
State Standards	9^{th} - 12^{th} Algebra II: 1 e
DOK Level	DOK 2 – Algebra II
DOK Application	Infer, Identify Patterns, Construct, Modify
National Standards	9-12: I: Representation
Graduate Research Element	This lesson will use digital logic circuits and binary number representation to demonstrate how computers perform calculations, specifically number sequences. My research is based on the partial reconfiguration of digital logic designs on FPGA chips.

Student Learning Goal:

After performing this lesson, students will be able to use a digital logic circuit to produce a mathematical sequence.

This lesson will address Mississippi 9-12 Mathematics standard: Algebra II 1e through the demonstration of mathematical sequences by learning about Boolean algebra and interacting with digital logic circuits and National 9-12 Mathematics standard I1 by requiring students to use an alternative method to represent a mathematical sequence, and.

State Standards: 9^{th} – 12^{th} Mathematics

Algebra II - 1e: Solve applications and problems in mathematical settings involving arithmetic and geometric sequences and series.

National Standards: 9^{th} – 12^{th} Mathematics

I (Representation): Create and use representations to organize, record, and communicate mathematical ideas.

Materials Needed (supplies, hand-outs, resources):

- Breadboards – Breadboards are a prototyping circuit board consisting of multiple sockets in grid that allows for circuit construction and deconstruction by inserting wires to connect different components "plugged into" the board. (see teacher notes for further information)
- Background Presentation (see sequence_logic_presentation.ppt)
- Circuit components (list provided in teacher notes)
- Worksheet (see sequence_logic_worksheet.doc)

Lesson Performance Task/Assessment:
Students will use Boolean algebra and digital logic components to construct a full adder circuit (a full adder circuit is used to add to binary bits together and produce a value and a carry out that can be fed into a second stage if desired). They will modify the inputs of the circuit to demonstrate the ability to produce a mathematical sequence as an output of the circuit.

Lesson Relevance to Performance Task and Students:
Students will design a digital adder circuit to demonstrate an understanding of Boolean algebra. This will allow students to interact with digital circuits and require them to think critically about how to create circuit configurations that produce the correct results. Students will then utilize the full adder circuit to generate various math sequences, such as the Fibonacci sequence, to show a useful application of Boolean algebra. This will give the students an opportunity to see how mathematical principles are found in real-world applications.

Anticipatory Set/Capture Interest:
The lesson should begin with the instructor showing the students an image of a computer motherboard similar to this: http://upload.wikimedia.org/wikipedia/commons/d/de/386DX40_MB_Jaguar_V.jpg. The instructor will then ask if the students recognize the image. Then, the instructor will show an image of an RTL circuit similar to this: http://adamwsonu.files.wordpress.com/2010/03/quizcircuit.png .
The instructor will then pose the same question. Next the instructor will explain that the images are conceptually the same and that computers work by performing math problems and pose the question "how do computers do this math?" This will lead into a discussion of Boolean algebra.

Guided Practice:
After the anticipatory set, the lesson will begin with a brief introduction to Boolean algebra (see sequence_logic_presentation.ppt for more information). The instructor will explain the basic logic concepts of "AND", "OR", and "exclusive or (XOR)" and how they can be combined together to produce varying results. The instructor will pose questions to the students using real world examples to assess the students understanding of the topic. The instructor may use the following questions to assess the students understanding:

- If I have one apple and one orange, do I have apples "**and**" oranges?
- If I have zero apples and one orange, do I have apples "**or**" oranges?
- If I have one apple and zero oranges, do I have "**exclusively**" apples "**or**" oranges?

Other variations of these questions can be used to evaluate true understanding of the concept of Boolean algebra.

The instructor will then split the class into teams of 3 to 5 students and distribute the breadboards (information on how to setup the initial state of the board and final circuit solution can be found in the included file: sequence_logic_circuit_information.doc).

The instructor will also distribute the wires and worksheets necessary to complete the lesson. The instructor will guide the students through the process of wiring the first two stages of the adder (see sequence_logic_presentation.ppt).

The instructor will then explain the independent portion of the lesson.

Independent Practice:
In the independent practice section of this lesson, students will work together in teams to wire the final two stages of the adder. The steps for building the circuit can be found in the included file: sequence_logic_circuit_information.doc.

Once the circuit is built, the students will show their understanding of mathematical sequences by using their circuits to generate the Fibonacci sequence and filling out a worksheet (see sequence_logic_worksheet.doc) explaining their process.

Remediation and/or Enrichment:
Remediation: Individual IEP; have instructor assist with circuit assembly.
Enrichment: Have the students use the circuit to produce a different mathematical sequence of their choosing and explain its output.

Check(s) for Understanding:
What do the switches represent with relation to a math sequence? What do the LEDs represent with relation to a math sequence? Based on the switches' current position, how do you know which led will light up? Why do you need more than one adder stage in the circuit?

Closure:
Discuss how simple circuits like the one created in this lesson are the basis for all computers today, the relevance of the Fibonacci sequence to computers, and how more complex mathematics concepts can be performed with larger versions of the same circuits. Finally discuss how digital circuits like the ones built in class can be used to create programs that can reconfigure themselves.

Possible Alternate Subject Integrations:
AP Physics – Introduction to logic circuits

Teacher Notes:
This lesson should be performed by the instructor prior to classroom introduction. A thorough understanding of digital logic circuitry is required to instruct this lesson.

Supplemental File List:

1. sequence_logic_presentation.ppt
2. sequence_logic_circuit_information.doc
3. sequence_logic_worksheet.doc

Circuit components required: (for one team)

1. Breadboard
2. (2) Quad 2-input AND gate – 7408
3. (2) Quad 2-input XOR gate – 7486
4. (1) Quad 2-input OR gate – 7432
5. (4) Red LEDs
6. (4) 1K ohm resistors
7. (1) 5V Regulator
8. (1) 9V Battery connector
9. (1) 9V Battery
10. Assorted Wires

Further information about the individual components can be found in the file: sequence_logic_circuit_information.doc.