Lesson Title	Capacitance
Length of Lesson	90 minutes
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Subject	Physics
Grade Level	11-12
State Standards	Physics: 5
DOK Level	DOK 4
DOK Application	Analyze, Draw Conclusions, and Develop a Logical Argument
National Standards	Physics B
Graduate Research Element	Data analysis, model fitting, estimating error, experiment design.

Student Learning Goal:

Mississippi Standards:
Physics: 5. Apply an understanding of magnetism, electric fields, and electricity.

National Standards:
Physics:
In some materials, such as metals, electrons flow easily, wheras in insulating materials such as glass, they can hardly flow at all. Semiconducting materials have intermediate behavior. At low temperatures some materials become superconductors and offer no resistance to the flow of electrons.

Materials Needed (supplies, hand-outs, resources):

- 1 operational amplifier
- 2 1KOhm resistors
- 1 approximately 500 Ohm resistor (470 Ohm works good.)
- 1 MOhm resistor
- 1 variable capacitor (Preferably one with parallel plates that the distance between them is adjusted.)
- 1 oscilloscope
- 1 power supply (3-12 volts, batteries work really good)
- 1 voltmeter (Possibly not needed if the oscilloscope is good enough.)

Lesson Performance Task/Assessment:
The students will have to measure the capacitance of the variable capacitor and relate it to the theory.

$$C = \frac{\epsilon_0 A}{d} \tag{1}$$

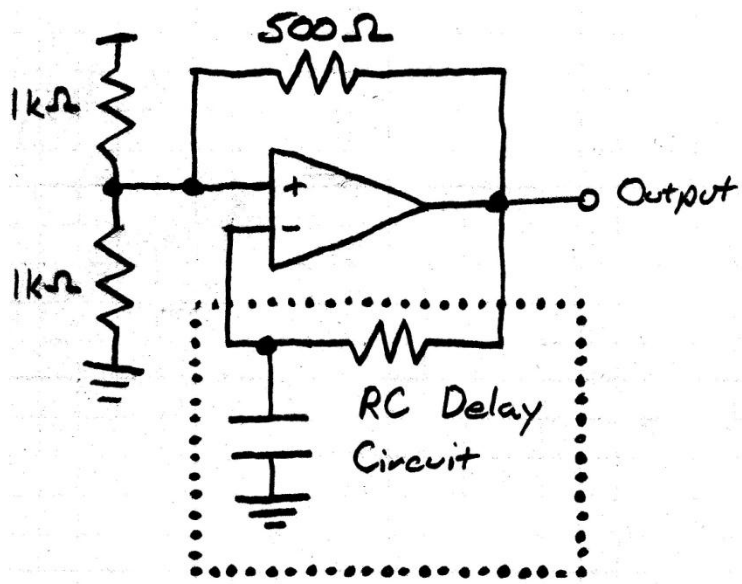
C = capacitance.

ϵ_0 = permittivity of free space = 8.854187817x10^{-12}
A = cross section area of the capacitor.
d = distance between the plates.

Lesson Relevance to Performance Task and Students:
The students will be measuring capacitance of a capacitor that they can see the insides of. They will using their measurements to verify the theory in their text books.

Anticipatory Set/Capture Interest:
Having a working version of the circuit at the front of the classroom to show them as soon as class starts.

Guided Practice:
Unless you have some exceptional students, you are going to have to talk to them about how the circuit works. Since electric circuits are not common knowledge, here is an explanation of the circuit. The circuit looks like the figure below.



Op amp:

There is an operational amplifier (op amp) in the circuit. It is used as a threshold device in the circuit. Ideally, op amps output the source voltage (that is the voltage of their power supply) when the positive input is greater than the negative input, and 0 Volts when the negative input is greater than the positive input. Ok, so let's look at what happens to the positive input first. We will assume we have an ideal op amp with a 6 Volt power source. When the output voltage is 0 Volts, the 500 Ohm resistor is in parallel with the bottom 1 KOhm resistor. When resistors are in parallel, the resulting resistance is computed like this:

$$R_p = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \tag{2}$$

So, the resistance of the 500 Ohm and 1 KOhm resistor together is:

$$R_p = \left(\frac{1}{1000} + \frac{1}{500}\right)^{-1} = \frac{1000}{3} \approx 333\Omega \tag{3}$$

When resistors are in series, the total resistance is the sum of the two resistances.

$$R_s = R_1 + R_2 \tag{4}$$

So the total resistance between the source and ground is 1333 Ohms ($1000 + 333$). We can use Ohm's Law to compute the current flowing from the source to ground at the positive input. (There is no current flowing in the inputs of an op amp by design.)

$$I = \frac{V}{R} = \frac{6}{1333} \approx 4.5\,mA \tag{5}$$

So, the voltage at the positive input is computed using the current and the resistor closest to the ground (in this case the 333 Ohm equivalent).

$$V = IR = (.0045)(333) \approx 1.5\,V \tag{6}$$

A similar process can be used to compute the voltage when the output of the op amp is 6 Volts (the 500 Ohm resistor will be in parallel with the 1KOhm resistor on the top). To save time I will give you the voltage, which is about 4.5 Volts. So, let's review the important stuff at this point.

- The output voltage of the op amp will be 0 Volts whenever the negative input voltage is greater than the positive input voltage. When the output voltage is 0 Volts, the positive input voltage will be 1.5 Volts.
- The output voltage of the op amp will be 6 Volts whenever the positive input voltage is greater than the negative input voltage. When The output voltage is 6 Volts, the positive input voltage will be 4.5 Volts.

Ok, now let's look at the negative input of the op amp. The input is the voltage stored on the capacitor in the RC Delay Circuit. When the output of the op amp is 0 Volts, the voltage on the capacitor discharges through the resistor. It approaches 0 Volts, but this continues only so long as the voltage of the negative input is greater than the voltage of the positive input (which we determined is 1.5 Volts when the output is 0 Volts). When the voltage on the capacitor gets below 1.5 Volts, the voltage of the positive input is greater than the voltage of the negative input, so the output voltage changes to 6 Volts. This causes the capacitor to start charging through the resistor. It approaches 6 Volts, but this continues only as long as the positive input voltage is greater than the negative input voltage (we know the positive input voltage is 4.5 Volts when the output voltage is 6 Volts). When the voltage on the capacitor is slightly greater than 4.5 Volts, the output voltage goes to 0 Volts. I hope at this point you can see that this is a repeating cycle, and the output voltage will cycle between 0 Volts and 6 Volts. (This is why the circuit is called an oscillator.)

If we know the value of the resistor in the RC Delay Circuit, we can estimate the value of the capacitor. The voltage on the capacitor will follow the equation:

$$V(t) = V_f + \left(V(0) - V_f\right)e^{-\frac{t}{RC}} \tag{7}$$

$V(t)$ is the voltage at time t, V_f is the forcing voltage, $V(0)$ is the initial voltage, t is time, R is the resistance in the RC Delay Circuit, and C is the capacitance in the RC Delay. If we can measure the period of the output voltage (remember it oscillates between 0 Volts and 6 Volts) say with an oscilloscope, then the time it takes to charge the capacitor is half the period. This means we know the value of all the variables in equation 7 except C, so we can solve for C. In our example, the values of the variables are as follows:

- $V(t) = 4.5$ Volts
- $V_f = 6$ Volts
- $V(0) = 1.5$ Volts
- t = half the period
- R = the value of the resistor (I recommend you give the students 1 MOhm resistors and have them measure the resistor to get a precise value.)
- C = unknown, but the students can solve for it.

In this explanation, I left out a few minor detail since I assumed the op amp was "ideal." Practical op amps have manufacturing flaws, so it is not possible to get the maximum and minimum output voltages to be exactly 6 Volts (in our case) and 0 Volts. This means the students will need to measure the minimum and maximum voltages for the output of the op amp and the positive input. This will change the numbers a little, but the students can measure the capacitance very accurately still.

Independent Practice:
The students will measure the capacitance of the capacitor using the circuit (equation 7), and compare their measurement to the theory (equation 1). You may want to have the students change the distance between the capacitor plates and measure the capacitor several times. You should decide how many times is sufficient. We had our students do it 10 times, but that may be a little high.

Remediation and/or Enrichment:
Enrichment: Have the students collect more data points.

Remediation: individual IEP; partner help throughout the lesson; the teacher can observe the students and intervene during the independent practice.

Check(s) for Understanding:
During the lab, the teacher can walk around and observe the students. If some of the students appear to not understand how the equipment works or what they are expected to do, ask them some leading questions.

Closure:
We closed by talking to the students about their lab reports.

Possible Alternate Subject Integrations:

Math

Teacher Notes:
I have included a video with this lesson showing how the lab equipment works.