Lesson Title	Averaging Data Demo
Length of Lesson	5-10 minutes
Created By	Matthew A. Lee, Henry Stauffenberg, and William Funderburk
Subject	Physics
Grade Level	11-12
State Standards	Physics: 1c, 1e
DOK Level	DOK 2
DOK Application	Make Observations, Identify Patterns, Relate
National Standards	
Graduate Research Element	Image Processing

Student Learning Goal:
This is a very short demo, but it illustrates to the students why they need to repeat experiments and average the results. It would work best if presented at the beginning of a semester or school year. Since it is so short, it can be incorporated into a lecture or a lab period.

Physics: 1c. Demonstrate the use of scientific inquiry and methods to formulate, conduct and evaluate laboratory investigations (e.g., hypotheses, experimental design, observations, data analyses, interpretations, theory development).
1e. Evaluate procedures, data, and conclusions to critique the scientific validity of research.

Materials Needed (supplies, hand-outs, resources):
PowerPoint.

Lesson Performance Task/Assessment:
This is a very short demo, so the assessment is not that obvious. The teacher can ask questions during the demo, but the demo will be much more effective if it is incorporated into a lecture or lab activity. Then the lecture or lab activity can have assessment devices built into it. For example, in a lecture, you could have a short quiz at the end of class and ask the students

- What effect does averaging data have?
- What does averaging noisy pictures show about randomness in our lab experiments?
- What sort of things make a science experiment more effective?

Lesson Relevance to Performance Task and Students:
The students will be able to observe how randomness is removed from images by averaging, and relate that to repeating experiments and averaging the results.

Anticipatory Set/Capture Interest:
Start the lesson by asking the students some questions. In the PowerPoint presentation, it starts off by asking "Why do we repeat experiments and average the results?"

Guided Practice:
The guided practice will consist of going through the PowerPoint presentation with the students. It walks the students through a very common experiment in signal processing college courses. However, it is a good way to show that random noise can be filtered by averaging. The PowerPoint presentation may leave a few things unclear, so a more in depth explanation follows here, but don't feel obligated to understand or explain everything to the students.

First off, we will start with a little math for a simple case. Every observation that we can ever make (even using the most expensive lab equipment available) can be modeled as the sum of the actual value and some random error associated with the way we make the observation. In math terms, that is:

$$observation = actual + error. \tag{1}$$

In advanced signal processing courses, the error is often modeled using a Gaussian (or normal) distribution that has a mean of μ and a standard deviation of σ. That is:

$$\mu = mean\ of\ the\ error \tag{2}$$

$$\sigma = standard\ deviation\ of\ the\ error \tag{3}$$

Now, I mentioned that the error is often modeled using Gaussian (or normal) distribution in advanced signal processing. That detail is not really important, but it is a common way of modeling random variables because of the *central limit theorem*, which just says that the distribution of sums of random variables always approach a Gaussian distribution as more and more random variables are added together. Now back to the important stuff…$\mu = mean\ of\ the\ error$, and $\sigma = standard\ deviation\ of\ the\ error$. In math and science, we generally estimate the mean of a random variable by computing its average.

$$average(x, y) = \frac{x+y}{2} \tag{4}$$

Or more generally:

$$average(x_{1-n}) = \frac{x_1+x_2+\cdots+x_n}{n} \tag{5}$$

So here is an important concept of this explanation: **averaging does not tell you μ, it is only a estimate of μ**. To some this will be a difficult concept, and if you don't want to explain this, it is fine because that is a PhD. concept and we are just teaching high school physics. (No sarcasm intended. Really, you can just sidestep that if you don't think it is relevant to your lesson.) So, for those teachers that are not sure what that concept means, let's say we have a random variable that we get from flipping a quarter, and we assign the value 1 to heads and -1 to tails. Everyone knows that for this random variable, $\mu = 0$. (In the long run, we get the same number of heads (1) and tails (-1), so the mean is zero.) Let's say we flip the quarter three times and get two heads and one tails…that is [1, 1, -1]. The average for our random variable is

$$average(x) = \frac{1+1-1}{3} = \frac{1}{3} \approx .333. \quad (6)$$

We could have gotten lucky (or unlucky) and gotten all heads, but in the long run we don't *expect* to always get heads (unless there is a trick coin, which means $\mu \neq 0$ after all). Most likely we will get something close to heads half of the time in the long run. Now, suppose we flip it 100 times and get 53 heads and 47 tails. The average would then be 0.06. Notice how this is getting closer to what we *expect* in the long run. So let's now take a step and define a function for what we *expect* the average to be in the long run, and call it $E(x)$.

$$E(x) = long\ run\ average = \lim_{n\to\infty} \frac{x_1+x_2+\cdots+x_n}{n} \quad (7)$$

If you are unfamiliar with this notation, it just means the average of an infinite number of repetitions. Now, let's discuss a few properties of this function. First, the expected value of any random variable is μ.

$$E(x) = \mu \quad (8)$$

Second, the expected value of a constant is always the constant.

$$E(c) = c \quad (9)$$

Third, the expected value of a sum of random variables is always the sum of their individual expected values.

$$E(x + y) = E(x) + E(y) \quad (10)$$

I hope you were able to follow all of that math. If you understand it, then you now have the tools to explain the basic case (equation 1). So let's think of the observation and the error as random variables, and the actual as a constant. Then, let's compute the expected value of the observation.

$$\begin{aligned} E(observation) = E(actual + error) &= E(actual) + E(error) \\ &= actual + \mu \end{aligned} \quad (11)$$

What this equation tells us is that if we repeat an experiment a large number of times and average the result, then the average will approach the actual value plus the mean of the error. So, if the mean happens to be zero, then averaging will eventually filter out all of the error. However, if the mean is not zero, then averaging will eventually make our estimates off by a constant.

Let's put this in terms of a physics experiment. Let's say I am trying to measure the time it takes a ball to fall 100 meters with a stopwatch. Now I can think of three random variables (ways to get error) in this experiment: releasing the ball and starting the stopwatch may not be at the same time (call this e_1), the air resistance might not always be the same each time (maybe there is wind) (call this e_2), and I might not be able to stop the stopwatch at the exact instant the ball hits the ground (call this e_3). Let's say I am not so proficient at operating the stopwatch and there is some constant reaction time between me seeing the ball dropped or hitting the ground and my brain telling my finger to push the button on the stopwatch. We can also assume that I don't always pay attention so well, so sometimes push the button a little early in anticipation, or a little late because I blinked or someone called my name. The air is doing its thing, but really, it is part of the system. If I wanted to eliminate the effects of the air, I should have done this experiment in a vacuum. So the observed time for each experiment would be given by the following equation.

$$observation = \left(t_{final} + e_2 + e_3\right) - (t_{initial} + e_1) \tag{12}$$

If we repeat this experiment a few times, then we approach the expected value, which is

$$\begin{aligned} E(observation) &= E\left(t_{final}\right) + E(e_2) + E(e_3) - E(t_{initial}) - E(e_1) \\ &= t_{final} + \mu_2 + \mu_3 - t_{initial} - \mu_1. \end{aligned} \tag{13}$$

Remember that μ_1 is the mean of the time it takes me to start the watch, μ_2 is the mean of the time due to air resistance, and μ_3 is the mean of the time it takes me to stop the watch. So, all of that error due to me not paying attention goes away. Furthermore, if my brain always takes the same amount of time communicating with my finger, then $\mu_1 = \mu_3$, so the equation reduces to

$$E(observation) = t_{final} + \mu_2 - t_{initial}, \tag{14}$$

and the only error left is the mean of the air resistance.

Again, all of this math is not necessary for the students to understand, but if you (the teacher) can understand it, then you should be able to answer any questions the students have about the demo. (Depending on the level of the students and the amount of class time, you might want to sidestep or simplify some of the explanation.) In the image in the PowerPoint, each pixel is an observation. I modified the image by adding a random value to the original image, so it is like equation 1. As the number of images are averaged together, all that is left is the actual values of the pixels in the image plus the mean of the random variable, which I ensured was zero. So, the key concept for the students is **averaging filters much of the noise in experiments**. If you can get them to understand that, then you are an A+ teacher.

Independent Practice:

There is not any independent practice in this lesson plan since it is just a demo, but if you want them to do something independent, you could have the students flip quarters or roll some dice and average the results. Of course, the time they spend on this would be added to the lesson time for this plan, but it would definitely increase the depth of knowledge. It is possible you could get a great whole class lesson by doing that, but it wouldn't be a physics lesson (maybe a math lesson).

Remediation and/or Enrichment:

Remediation: individual IEP; partner help throughout the lesson; the teacher can observe the students and ask questions.

Check(s) for Understanding:

Ask the students some questions. Some good questions are:

- What effect does averaging data have?

- What does averaging noisy pictures show about randomness in our lab experiments?
- What sort of things make a science experiment more effective?

Have the students demonstrate their understanding by designing labs that repeat parts and average the results. If they put repetitions in their lab designs, they probably got the point.

Closure:
Again, you can close by asking questions, or this can be part of your lesson for the day and you may want to close by focusing on the rest of your lesson.

Possible Alternate Subject Integrations:
Math, Computer Programming

Teacher Notes:
There is a PowerPoint presentation with this lesson called "Effects of Averaging Data.pptx." Feel free to modify it or redistribute it as you wish as long as you credit the original author some way.

Also, some of the students will be confused by the term "noise" because they associate it with hearing only. It might be better to use the word "randomness" or explain their misconception.