Lesson Title	The Information in the Spectrum of Light
Length of Lesson	2 days
Created By	Matthew A. Lee, Henry Stauffenberg, and William Funderburk
Subject	Physics
Grade Level	9-12
State Standards	Physics: 1d, 1g, 4e Algebra II: 2b
DOK Level	DOK 3
DOK Application	Use concepts to solve non-routine problems
National Standards	9-12: B (Physical Science)
Graduate Research Element	Signal Processing

Student Learning Goal:

This would be an excellent lesson to follow up a study of the Bohr model of Hydrogen and the Balmer series.

The students will learn how to use the electromagnetic spectrum to estimate quantitative information about the composition of objects. They will use diffraction gradients to observe the spectra of visible light emitted by excited gases. Also, the students will use a hyperspectral sensor to measure the spectrum of visible light as well as invisible portions of the electromagnetic spectrum. They will then use these measurements and their ability to solve linear systems of equations to estimate the composition of an unknown substance.

Physics:
1d. Organize data to construct graphs (e.g., plotting points, labeling x-and y-axis, creating appropriate titles and legends for circle, par, and line graphs) draw conclusions and make inferences.

1g. Collect, analyze, and draw conclusions from data to create a formal presentation using available technology (e.g., computers, calculators, SmartBoard, CBL's, etc.)

4e. Investigate and draw conclusions about the characteristics and properties of electromagnetic waves.

Algebra II:
2b. Solve systems of absolute value and quadradic equations using a variety of solution methods including graphing.

National Science Education Standards of Content 9 – 12
B (Interactions of Energy and Matter)

- Electromagnetic waves result when a charged object is accelerated or decelerated. *Electromagnetic waves include radio waves (the longest wavelength), microwaves, infrared radiation (radiant heat), visible light, ultraviolet radiation, x-rays, and gamma rays.* The energy of electromagnetic waves is carried in packets whos magnitude is inversely proportional to the wavelength.
- *Each kind of atom or molecule can gain or lose energy only in particular descrete amounts and thus can absorb and emit light only at wavelengths corresponding to these amounts. These wavelengths can be used to identify the substance.*

Materials Needed (supplies, hand-outs, resources):

Gas-filled tube lamps, diffraction gradients, Excel, and a hyperspectral sensor.

A hyperspectral sensor measures the electromagnetic spectrum at hundreds of evenly spaced wavelengths. They always measure the visible spectrum, but often they measure ultra-violate, visible, and infrared. An example of a good handheld sensor to use is the FieldSpec® 3 Max Portable Spectroradiometer by Analytical Spectral Devices Inc. It has a spectral range of 350-2500 nm, and a sampling interval of 1.4 – 2.0 nm. This sensor is not cheap! It will probably cost nearly $100,000.00, so you may not want to let students handle it too much. Getting access to such a sensor may make this lesson not feasible for most high school classes, but you may be able to work with a nearby university to get access to one as the university may see it as a good outreach and recruiting opportunity. You might find one in the electrical engineering department, geosciences department, agriculture department, or chemistry department. A good way to start is by finding out if anyone specializes in remote sensing or signal processing, and ask them. Another possibility is to use hyperspectral signatures that have already been captured (see the teacher notes).

Lesson Performance Task/Assessment:

The students are going to be given known spectral signatures of pure gases, and an unknown spectral signature. These signatures will contain hundreds of samples at various wavelengths. Some of the wavelengths will correlate will particular gases. It will be the job of the students to estimate the percentage concentration of known gases in the unknown sample.

The unknown sample can be created mathematically by linearly combining the known samples. This can be done using the following equations.

$$\bar{u} = a_1\overline{k_1} + a_2\overline{k_2} + \cdots + a_n\overline{k_n} \tag{1}$$

$$1 = a_1 + a_2 + \cdots + a_n \text{ for all positive } a_i. \tag{2}$$

The bar indicates the variable is a vector. In this case it is the spectral signatures of the gases. This process does accurately model physical realities in certain common situations I have experienced in my research. For example, consider that the hyperspectral sensor has a field of view that is shaped like a cone. The sample is typically placed on a planar surface some distance from the sensor, so the sensor sees an area on that plane that has the shape of an ellipse (usually it is close to a circle). If the sample takes up 50% of that area, then the sensor will detect a spectrum that is 50% sample and 50% background. This is referred to pixel mixing in the remote sensing community.

We will be asking the students to do something called pixel unmixing, which is an active research area with many unsolved problems. However, we will trivialize the task by not dealing with noise and ensuring they have pure known hyperspectral signatures to work with. They will do this by first picking a set of wavelengths to work with ($w_{1\ to\ n}$) from the hyperspectral signatures. They will then solve a system of equations with the following template.

$$a_1\overline{k_1}(w) + a_2\overline{k_2}(w) + \cdots + a_n\overline{k_n}(w) = \bar{u}(w) \quad (3)$$

Of course, since we have n wavelengths (w), we will have n equations.

Lesson Relevance to Performance Task and Students:
Students should be able to explain how to estimate the composition of unknown samples using the emitted electromagnetic spectrum. They should understand that light is a subset of the electromagnetic spectrum. They should get some practice solving systems of equations. They will observe that sometimes you can have more data than you need to solve a problem.

Anticipatory Set/Capture Interest:
This should be easy. You will be giving them a light show with the various gas filled tubes and diffraction gradients, and showing off really expensive and sensitive equipment (the Analytical Spectral Device).

Guided Practice:

First, the teacher should give the students some background about the Balmer series. It will be useful for them to understand how hydrogen creates the Balmer series pattern, but it is not absolutely necessary. It is necessary that the students realize that the spectrum of light emitted, absorbed, and reflected by matter is unique to its molecular composition.

Second, the teacher should pass out diffraction gradients and turn the lights out so that the students can see Balmer series in a Hydrogen filled lamp. If a hyperspectral sensor is available, the teacher can measure the spectrum from the Hydrogen lamp. If a hyperspectral sensor is not available, the teacher can show them pre-measured plots. In any case, the plots will correlate to what the students see. Plus, the students will see that the light intensity is not the same at all bands in the Balmer series. This is a fact that is not obvious to the eye when looking through a diffraction gradient. Also point out that the spectrum measured by the hyperspectral sensor contains ultra-violate, visible, and infrared. The students may also notice noise in the plots. If this happens, talk to the students about possible sources of that noise, such as other light sources, sensor deficiencies, not holding the sensor and lamp stationary, or perhaps the flickering of the lamp.

Third, the teacher and students should observe and measure the spectra from different gases. This should help the students understand that the spectrum is unique for different materials. To reinforce this even more after the class finishes looking at the gases, the instructor could turn on the lights, white reference the sensor, and take reflectance readings of different materials (plants, skin, hair, clothes, etc.). This would be a good point where the class could become interactive. One thing the teacher should point out is that materials that are made from plants usually have a spectrum that increases dramatically as it goes from red wavelengths to IR wavelengths. This is called a red edge, and it can be seen in unpainted wood products, cotton shirts, some dyes, leaves, grass, etc.

Fourth, in this section the teacher will teach the students how to do pixel unmixing. The teacher will talk about how the spectral signature is usually a mixture of multiple spectral signatures, and we can estimate the proportion of each spectral signature if we know how many signatures and what a pure signature of each source looks like. The teacher can then talk a little about solving systems of linear equations. After this review, the teacher should show the students a signature that was created using the linear mixing process with two gases (or you could use some other signatures such as plant and soil). The teacher should talk to the students about the fact that we have two unknown variables to solve for (thus we only need the light intensity at two different wavelengths), but we have hundreds of samples in the electromagnetic spectrum. Ask the students "what we should do?" The answer is that we can pick any two wavelengths we want and come up with an estimate. Ask "what if there is noise in the signal?" (There is noise even if it is just round off error.) "How can we get the best estimate of the gas proportions?" "Are there some wavelengths that will yield better estimates?" The simple answer is that the wavelengths that contain the greatest difference between the two pure signatures are the best. (Also avoid parts of the spectrum where there is a lot of noise.) The teacher should then work through the solution choosing the best wavelengths to estimate the concentrations. The teacher can repeat this for a mixture of three or more signatures to show that the number of unknown variables and equations in the system increase.

Independent Practice:

The students should be given an unknown mixed signature with multiple gases (or other materials), and asked to estimate the proportion of each material in mixed signature. They will need to know what the pure signatures are as well as the unknown signature. A good way to give them this information is in an Excel file. They can then graph the known signatures to help them choose what wavelengths they will use. If the unknown signature is composed of too many pure signatures, it may be necessary for them to do it as homework.

Remediation and/or Enrichment:

Remediation: individual IEP; partner help throughout the lesson; the teacher can observe the students and intervene during the independent practice.

Check(s) for Understanding:

At various points in the lesson, the teacher should ask the students questions.

There are many ways the students can struggle in the independent practice, and the teacher can check for these. A few possible ways are listed below.

- The students may fail to choose an appropriate number of equations. This shows they do not know how many unknown variables there are, or they do not realize they need the same number of equations as there are unknown variables. They should have an unknown variable for every pure signature that is mixed in the unknown signature.
- The accuracy of the students' results may be poor. This could be caused by not picking appropriate wavelengths or simple math errors.

Closure:

The teacher can close with a discussion about fields of technology that use the fact that you can identify the composition of objects based on their electromagnetic spectrum. A few good fields are:

- Remote Earth sensing. How much ozone/smog/pollution is in the atmosphere? How much forest/agriculture/urbanization is there on the ground?
- Space exploration. What is a nebula/star/planet made of? Is there water on that planet/comet/asteroid? Is it possible there could be carbon based life on something?
- Construction. Are there impurities in these building materials? Is that concrete dry enough?
- Chemistry. What is in this sample?

Possible Alternate Subject Integrations:

There is definitely some good math in this lesson.

Teacher Notes:

When making this lesson, I realize that I am extremely fortunate to have access to a portable hyperspectral sensor, and most teachers will not have access to such a device. Therefore, I am including an Excel file with this lesson that has some signatures of gases. While I think it is definitely worth it for the students to see how the signatures are collected from the samples, they will still benefit from the lesson even if they can't see this demonstrated.

Also, as I said, pixel unmixing is an area that is currently being researched. The main unsolved questions in the area are as follows.

- How do you know how many pure signatures are in a mixed signature?
- How do you deal with noise in both the pure signatures and the mixed signature?
- How do you know if you have a pure signature? Very often in remote Earth sensing applications you have an image created by sampling a landscape with a hyperspectral sensor in a grid pattern. This is often done by a satellite where

each pixel is approximately 30x30 meters. Thus getting a 100% pure signature is almost impossible.

- What do you do when you don’t have a pure signature?

If you get an ambitious student, they could potentially get a publication in a research magazine if they can come up with a new way to attempt to solve one of these problems.

I did part of this lesson with my class, but I was not able to do the pixel unmixing part because of time constraints. I am including my short PowerPoint presentation that I used with the lesson. It is in the file called "Information in the Spectrum." I hope this helps.