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| **Lesson Title** | Building Paper Bridges |
| **Length of Lesson** | 1 day |
| **Created By** | Kylie Nash and John DuFour |
| **Subject** | Math |
| **Grade Level** | $9^{th}$ -$12^{th}$ (Geometry, Algebra II) |
| **State Standards** | Geometry: 1a; 5a, b and Algebra II 5 a (see extension section) |
| **DOK Level** | DOK 2 Geometry |
| **DOK Application** | DOK 2 – Identify, Relate, Construct, Use Contest Clues, Make Observations, Predict, Show |
| **National Standards** | $9^{th}$- $12^{th}$ Geometry, Algebra |
| **Graduate Research Element** | Engineering Design Concepts- Increasing conceptual knowledge by demonstrating engineering concepts of design using shapes and relating weight properties |

**Student Learning Goal:**

State Standards for $9^{th}$ – $12^{th}$ Geometry: Geometry

1a) Apply problem solving skills to solve and verify the solutions for unknown measures in similar polygons.

5a) Draw conclusions and make predictions for scatter plots, b) Use linear regression to find the line-of-best fit from a given set of data.

National Standards for $9^{th}$ -$12^{th}$ Geometry and Algebra Standard:

- Use geometric models to gain insights into, and answer questions in other areas of mathematics.

Students will be able to be able to practice and use appropriate techniques to solve mathematical problems linked to real world engineering concepts. Construct 3D shapes and identify relationships and make predictions on the optimal solution for solving problems.

**Materials Needed (supplies, hand-outs, resources):**

Writing utensils, 1 sheet of paper, 5 paper clips and two books of equal measurement and enough pennies per group (at least 100 per group), projector for presentation; guided information about topic and guidelines in PowerPoint presentation by instructor.

**Lesson Performance Task/Assessment:**

Students will build two styles of bridge demonstrating their understanding of various shapes and the strengths of those various shapes. Students will be able to practice constructing 3D shapes, understand the symmetry of the design, and predict how much weight the different shapes are able to hold.

**Lesson Relevance to Performance Task and Students:**
Allowing students to construct bridges based on their design ideas will help build their critical thinking skills and enhance their problem solving skills for determining how to make the structure able to hold the most weight. Simulating real world applications will help strengthen math skills and engineering design concepts to students as well as how math, specifically geometry can be applied to real world applications. Increase understanding the function follows form of how different geometric shapes and designs are stronger than others. Through the use of hands on activities to synthesize and interpret concepts learned in the classroom.

**Anticipatory Set/Capture Interest:**
Students will be divided into groups of 3 to 4 and told that they are part of a team of civil engineers and have been selected to participate in a contest to build the cities next bridge. The team with the best design, able to hold the most weight will win the contest and their bridge will be built in the city with their names written on the bridge. Students will then be introduced to different bridge styles by showing photos of existing bridges and showing different construction types as well as what kinds of conditions need to be considered when building a bridge.

**Guided Practice:**
The instructor will give an introduction to types of bridges and discuss how the weight is distributed on the various types of bridges based on the particular shape of the bridge. Then the instructor will pose questions concerning engineering design considerations for various structures, symmetry, shapes and weight distribution of the different shapes. The instructor will discuss how this application can be applied to other real world applications. The instructor will also lead the discussion using the questions listed below:

1. How many of you know what a civil engineer or structural engineer does?
2. What shapes are used to build bridges?
3. What type of terrain is involved with a bridge?
4. Required clearances. Do boats need to go under the bridge?
5. How many lanes? What will go over the bridge?
6. What materials/technology is available?
7. Environmental and Weather related conditions and impact.

The instructor will then provide the guidelines and rules for building the bridge described below.

Guidelines:
- Bridge must span 8.5 inches.
- Nothing will go under the bridge.
- The bridge must span 2 books.
- The bridge must hold 100 pennies.

**Rules:**
- You may fold, wrap, crease, or roll your sheet of paper in any way.
- Paper clips may only touch the paper bridge not the books.
- All pennies must be between the two books.
- The paper may only touch the 2 books.

The students will be instructed to build their bridges based on the above guidelines and rules. Each team will add pennies to their bridge one at a time until the bridge collapses. Record the number of pennies the bridge held until it collapsed. After the students shave built their bridges then the instructor can lead the discussion on why their bridges collapsed as well as how many pennies each held before it collapsed. The instructor or student can report each team's number of pennies and folds on the board, to keep the completion going.

Many students should not be able to hold 100 pennies. The instructor should introduce the idea of the accordion style model for the bridge. The instructor should reinforce the idea of how triangles are the strongest shapes and how several together are even stronger. Most students will identify with making paper fans made in childhood rather than an accordion. Then the instructor should instruct them to build a new bridge based on the fan style. The goal is to hold more than 100 pennies and count the number of folds. The paper must be folded on the long edge and in order to have 4 fold the paper must be folded 5 times.

**Students can be given several sheets of paper to keep trying their designs, but check to make sure only one sheet is being used at a given time for both parts of the activity.

**Independent Practice:**
A. Students will divide into groups and first build bridges based on any style and shape they choose, using one of the initial 4 shapes described by teacher. Each team will add pennies to their bridge one at a time until the bridge collapses. Record the number of pennies the bridge held until it collapsed. Students will discuss why their bridges collapsed with the number of pennies it held. They should recognize that their bridge structures don't have an even distribution of support. The winner can be determined by calculating the penny to fold ratio, and the team with the highest ratio wins.

B. Once the students have been introduced to the accordion style bridge, then they will build new bridges based on these criteria to determine the max number of folds holds the most pennies. The teams for the groups can be kept the same as the first part of the activity and they compete to win for the highest number of folds and pennies. The teams may also be grouped into specific teams shown below depending on time not to determine a winner or get a prize. Either team will add pennies to their bridge until the bridge collapses and report the number of pennies and number of folds the structure held before it collapsed.

Make bridges using the accordion style to determine max number of folds and pennies based on non-competing teams:

- Team 1: 3 fold bridge
- Team 2: 6 fold bridge
- Team 3: 9 fold bridge
- Team 4: 12 fold bridge
- Team 5: 15 fold bridge
- Team 6: 18 fold bridge
- Team 7: 21 fold bridge
- Team 8 will find the maximum number of accordion folds that a single sheet of paper can have.

They will add pennies to their bridge until the bridge collapses and report the number of pennies and number of folds the structure held before it collapsed.

**Remediation and/or Enrichment:**
Remediation:
Individual IEP, shorten activity and let students build bridges using only either only the 4 types of bridge designs or the accordion style and discuss the reasons for success or failure, individual assistance. They will add pennies to their bridge until the bridge collapses and report the number of pennies or number of folds the structure held before it collapsed.

Enrichment/Extension 2nd Day:
State Standards for 9th – 12th Algebra II: Data Analysis & Probability
DOK 3 Algebra II (extension)
5a) Through the use of technology, use scatter plots and linear and quadratic regression analysis to determine an appropriate function to model real life data.

Introduce Algebra II concepts, use scatter plots and linear and quadratic regression analysis to determine an appropriate function to model real life data. Let students collect data to and teacher will create scatter plots of the data to look at patterns from the accordion style 8 team using 3 tests data points.
x axis folds
y axis=pennies

| team | # of folds | test 1: pennies held | test 2: pennies held | test 3: pennies held |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |

Analyzing the original scatter plot shows that an accordion design provides a diminishing return.

- See that there is a maximum number of folds, after which, the bridge begins to weaken as folds are added.
- The function has a maximum value.
- At a point, as folds increase, the paper becomes more flat!

Next, the students can enter the data into their calculator and complete the following regression analyses.

- Quadratic regression; generate an equation.
- 3rd degree regression; generate an equation.
- 4th degree regression; generate an equation.
- Logarithmic regression; generate an equation.

The regression equations can be plotted here:

- http://graph.seriesmathstudy.com/graphictool.htm

The instructor should ask the students questions and lead the discussion in investigating these questions.

- Which equation best models the collected bridge data?
- This best equation can now be used to make future decisions about accordion paper bridge building.
- In this fashion, engineers generate formulas that are used to guide their work!

Then the instructor should ask:

- How can we modify our equation to increase safety?

Answer:
Lowering the value of the constant term or our equation will add safety. Lowering the value of the constant term ensures the bridge will actually hold more pennies than our equation implies. This is called “engineering with redundancy.”

Discussion:
Regression plots can be used to create math models for many things in real life.
Math models can be used to estimate behavior or make future predictions.

**Check(s) for Understanding:**
Day One:
1. Discuss the connections between styles of bridges and weight.

2. Discuss the relationship between number of folds and numbers of pennies the object can hold. Did the pennies work better in the center or evenly?

3. Discuss the differences between the 4 traditional style bridge types and the accordion style design

4. What shape is the strongest?

5. What parts of the presentation and activity did you feel was the most important to helped create the 3D model of the bridges and why?

6. What parts of the presentation and activity did you feel was the least important to helped create the 3D model of the bridges and why?

**Closure:**
Discuss some real world applications (who, what, when, where and how) that would benefit from understanding concepts related to geometric shapes and strength of bridges design. Discuss the relationship between the number of folds and the amount of weight compared to other bridge shape designs.

**Possible Alternate Subject Integrations:**
Physics- Look at what role such as tension and forces play on the strength of different polygons shapes and designs. Explore the impact of gravity on the bridge structure.

**Teacher Notes:**
**Reference Sources:**
1. http://www.learnnc.org/lp/pages/3050
2. http://www.pbs.org/wgbh/buildingbig/educator/act_paper_ei.html
3. http://www.sciencebuddies.org/science-fair-projects/project_ideas/CE_p018.shtml